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A MULTIPLE-PRODUCT SALES FORCE ALLOCATION MODEL*

by
David B. Montgomery** (Bruce), Alvin J. Silk*** (John),
and
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517-71 March, 1971

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ABSTRACT

When several products are marketed by the same sales force, it frequently becomes impossible or impractical for salesmen to promote all items in the product line extensively in each and every time period. Management's problem is to decide how the available selling effort should be allocated across products and over time. The opportunity costs associated with using limited selling resources to promote certain products but not others must be evaluated. This paper describes a decision calculus-type modeling system for dealing with this question.

The problem is analyzed by a two-step procedure. First, a response function is defined which relates selling effort to sales and profit results in a manner which represents some behavioral phenomena considered to be important. An interactive conversational program elicits judgemental data from managers which are used to parameterize the response model. A separate response function is specified for each product in the firm's line by this method. The set of response functions so obtained becomes the input for the second component of the system, an allocation heuristic. An incremental search procedure is employed to find an allocation of the sales force's time to the various products and over several time periods which is "best" in terms of total contribution to company profits. The model is presented in the context of an ethical drug manufacturer's multiple-product sales force allocation problem. Results of an application are summarized and implementation considerations noted. A comparison of the model-based allocation with that determined previously by management indicated that the former plan would offer a substantial improvement in profits.

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1. INTRODUCTION

Most of the management science research reported to date on problems of personal selling has been concerned with some type of sales effort allocation decision. Given that the sales force available to a firm for some short-term planning period is typically a fixed and scarce resource, the basic management question to be answered is, how should the salesmen be utilized in order to maximize profits? In their recent review of this work, Montgomery and Urban [15] discuss a variety of models for allocating sales effort among customers and prospects, geographical areas or territories, and time periods (salesman scheduling and routing). This paper describes a model designed to deal with the problem of allocating selling effort along yet another dimension--across a firm's product line.

Firms marketing several products through a single sales force frequently find it either impossible or impractical to have their salesmen promote the entire product line extensively in each and every time period. Clearly, there is an opportunity cost associated with using the sales force to promote certain products while withholding this support from others. When a firm has numerous products and personal selling is a key element in its marketing mix, the effect on total profits of different allocations may be very significant and evaluating the trade-offs implied by alternative policies is not a simple task. Organizational factors may further complicate this allocation decision, especially in firms where the products are distributed among a group of managers for purposes of marketing planning and control. Faced with the well-known obstacles to measuring market response to selling effort accurately, it is scarcely surprising to find managers relying more on bargaining than analysis to resolve conflicts about how the sales force's limited time should be divided up among competing uses.

The system for allocating selling effort among a set of products to be presented here represents an example of the type of model which Little has labelled a "decision calculus"--"a model-based set of procedures for processing data and

judgements to assist a manager in his decision-making" [13]. Two sub-models make up the system. First, a response function is defined which relates selling effort to sales and profit results and represents some key behavioral phenomena involved in the process. Estimates of the parameters for the response model are derived from judgements supplied by product managers with the assistance of an interactive, conversational program. A separate response function is specified by this procedure for each product in the firm's line. This set of response functions comprises the input required by the second component of the system, an allocation heuristic. The latter employs an incremental search procedure to find a "best" allocation of the sales force's time to various products over a number of time periods. The specific problem context for which this system was developed is that of an ethical drug manufacturer. The overall approach and model structure are, however, applicable to allocation problems of a similar nature found in other settings.

2. PROBLEM DESCRIPTION

2.1 General Considerations

A brief outline of the sales force allocation problem of interest here will serve to define the basic assumptions of the model and establish the nature of the constraints which a solution must satisfy.

(1) Firm. A firm sells a set of products in a competitive market. Product line interdependences are unimportant and can therefore be ignored. In economic terms, it is assumed that cross elasticities of demand with respect to sales effort for the firm's products are zero. Personal selling is a primary tool for promoting the product line.

(2) Customers and Prospects. Past and Potential buyers are numerous and heterogeneous in the sense that each typically needs and/or uses some but not necessarily all of the products sold by the firm. Those responsible for purchase decisions are willing or able to give salesmen only a limited amount of their time and hence, the duration of a sales call is ordinarily shorter than the seller would like.

(3) Sales Force. For the planning horizon under consideration (e.g., a year), the size of the sales force which management has at its disposal is essentially fixed. Each salesman operates in an exclusive territory. The major portion of his compensation consists of a fixed salary. Additional bonuses or commissions which he may receive depend upon some overall performance measure like total sales rather than the sales or profitability of particular products in the firm's line. A salesman can contact only a fraction of the customers and prospects in his territory in any time period and the number of products he can promote in any sales call is also restricted because buyers will only tolerate brief sales presentations.

Given the size of the firm's product line and the constraints on the size of the available sales force and the number and length of the calls each salesman can make, all products cannot be promoted to all prospective customers in all time periods. Hence, there is need for a method of determining how the sales force's efforts should be utilized in order to "maximize" total company profits.

2.2 Detailing Ethical Drugs

The general type of sales force allocation problem described above arises in a number of diverse marketing contexts. To date however, our work has focused on the operations of a manufacturer of ethical drugs. Salesmen in this field are known as "detailmen." Their principal function is to promote drugs to physicians who, in turn, write prescriptions for their patients. A sale occurs when a retail pharmacy fills a prescription. Detailing is invariably the largest item in the promotional budget for ethical drugs. Evidence developed from numerous surveys conducted over the years indicates that detailmen are widely perceived by physicians as being legitimate and influential sources of information about drugs [1].

In the firm studies here, detailmen tour through their territories once every period (e.g., a quarter). A detailman calls on N different physicians in this period. Territories are designed so that this figure is approximately equal for all detailmen. Recognizing that the physician-detailman encounter will ordinarily be brief, the company has come to believe that no more than three products can be effectively presented in a single sales visit. The key decision variable

in planning the allocation of detailmen's time is the number of physicians to whom a product should be promoted in a given six week period. While theoretically it is possible for a product to be detailed to any number of physicians between zero and N, as a practical matter only four discrete alternative policies are considered feasible. These alternatives are listed in Table 1.

Insert Table 1 About Here

Applying a detailing policy of "complete coverage" to a product means that it is promoted to all N physicians that each detailman calls on in a given period. Such extensive detailing is primarily used for new products. The principal occasion for employing this policy with an established product is when the firm feels that it has something new and of significance to tell physicians, such as the findings of an important clinical study. As a result of the specialized nature of their training and practices, physicians differ markedly in their potential need for and use of various classes of drugs. It becomes desirable therefore, to detail drugs selectively and the "half" and "quarter" coverage policies are intended to accomplish this. Half-coverage is the usual method of detailing applied to mature products and results in a drug being promoted in fifty per cent of the calls made by detailmen. Quarter-coverage gives a product exposure to N/4 doctors. Use of quarter-coverage became necessary as a consequence of growth in the size of the product line. The firm's management believes that more specialized policies could not be implemented successfully.

It may be readily seen that the choice of detailing policies affects the total number of products which can be detailed in any period. Since each detailman contacts N doctors and promotes three products in each call, the amount of detailing effort available is 3N "product calls." The above table shows the number of calls required by each detailing policy. Let the number of products

TABLE 1

Detailing Policies

Detailing Policy (X_t)	No. of Calls/Period	REV (X_t) = $f(X_t)$	
Complete-Coverage	N	c	(c > 1)
Half-Coverage	N/2	1	
Quarter-Coverage	N/4	q	(0 < q < 1)
No Detailing	0	0	

receiving "complete," "half," and "quarter" coverage be represented by D(C), D(H), and D(Q), respectively. The values of these latter quantities must satisfy the following equation:

$$3N = D(C)N + D(H)N/2 + D(Q)N/4 \qquad (1)$$

The number of products that it is possible to detail in any period may vary from a minimum of three if all received "complete" coverage to a maximum of twelve if "quarter" coverage were employed exclusively. Currently, the detailing force studied here handles ten distinct products.

For each time period a plan is prepared which establishes which products are to be detailed and what type of coverage each is to receive. The target audience to which detailing efforts are to be directed is identified by the provision of guidelines which assist the detailman in deciding which physicians he should contact. However, the detailmen themselves select the particular doctors they visit since they know a great deal about the physician population in their territories.

The detailing plan also specifies the content of the sales message to be used by the detailmen in promoting a product. Detailmen's appraisals of the qualitative value of such material is particularly significant because unless they are convinced that they have something meaningful to communicate to physicians, it is unlikely that a policy will be carried out. Detailmen are understandably reluctant to jeopardize their relationship with a physician by taking up his time with familiar or unimportant matters.

3. THE RESPONSE MODEL

The method followd in modeling the allocation problem described above is that suggested by Little's concept of a "decision calculus" [13]. Motivated by by disheartening fact that only a small fraction of the models developed by management scientists are every actually implemented, Little has suggested some

desiderata for designing models that managers can use and will find useful. He argues that to meet a manager's needs, a model should be simple, robust, easy to control, adaptive, as complete as possible, and easy to communicate with.[2]

A decision calculus approach was especially appropriate in this context for two basic reasons. First of all, it was clear that any formal method for allocating detailing would have to be well understood by the product managers if there were to be any hope of having the results acted upon. The crux of the problem is to find a detailing plan that is "best" for the entire line rather than particular products comprising it. The inevitable consequence is that some product managers must settle for less detailing support for their products than they might otherwise consider desirable. Certainly, a minimum condition necessary for a manager to accept such an outcome is that he comprehend how the allocation decision was made.

Secondly, prospects for obtaining data-based estimates of sales response to detailing for <u>each</u> drug sold by the company were extremely limited. Historical measures of how much detailing various products had actually received were not available and would be difficult and expensive to collect on a continuing basis. Hence, statistical analyses of historical data did not appear to be a viable means of obtaining estimates of response parameters that would be satisfactory from both a technical and managerial standpoint. Experiments could be conducted bot not with more than a few of the firm's products in any one to two year period.[3] Heavy reliance would, therefore, have to be placed upon judgemental estimates of detailing impact and this could be expected to be a more or less permanent aspect of the allocation problem. What was required then was a system that would make use of the product managers' judgements and be well enough understood by them so they would feel comfortable using it.

3.1 <u>Exposure Value of Detailing</u>

In order to represent the relationship of detailing effort to sales results,

we postulate a two-stage response process similar to that employed by Little and Lodish in their media selection model [12]. Detailmen call on physicians and "expose" them to certain of the firm's products. The level of exposure maintained for a product among doctors affects their prescribing behavior and thereby sales. We begin by defining the "relative exposure value" of various detailing policies.[4] Let:

$$REV(X_t) = f(X_t) \tag{2}$$

where:

$REV(X_t)$ = Relative Exposure Value of a particular detailing policy.

X_t = A discrete detailing policy alternative used in time period t, i.e., X_t = complete coverage, half-coverage, quarter coverage, or no detailing.

Relative Exposure Value is a hypothetical construct scaled in the manner indicated in Table 1. That is, we measure REV on a ratio scale and arbitrarily assign the values of zero and unity to the policies of no detailing and half coverage, respectively. The value of quarter coverage naturally falls between zero and one while the impact of complete coverage will presumably be greater than that of half coverage. The parameters c and q can be assigned values so as to represent various patterns of scale effects with respect to detailing coverage. Since half coverage was the detailing policy managers regarded as most "normal," it was advantageous to set its REV equal to one and have it serve as a standard of comparison for judging the effectiveness of complete and quarter coverage.[5]

The exposure level that exists for a product in any period depends upon the amount of detailing support it has received in previous periods as well as the current one. However, with the passage of time, exposures will be forgotten and the level of REV will diminish. To account for these processes of accumulation and decay of REV, we employ an exponential forgetting function:

$$ATE(t) = \lambda f(X_t) + (1 - \lambda)\, ATE(t-1) \tag{3}$$

$$0 < \lambda < 1$$

where:

ATE(t) = level of REV in period t that has accumulated as a result of current and past detailing.

λ = forgetting parameter.

3.2 Sales Response to Accumulated Total Exposure

In this section we consider the linkage between accumulated total exposure in a period and the corresponding sales results. Sales in period t are given by

$$S(t) = SP(t) * SI(t) \qquad (4)$$

where:

S(t) = unit sales in period t.
SP(t) = sales potential in period t.
SI(t) = sales index in period t.

Sales potential refers to the sales which would be made in period t if the product has been heavily promoted in the past - i.e., if promotion has essentially saturated the market. The sales index is the fraction of sales potential which will be realized as a result of actual current and past sales effort which the product has received. Since current and past sales effort on behalf of a product are summarized in accumulated total exposure, the sales index (or fraction of potential realized) will depend upon the level of ATE.

The relation chosen to link ATE to the sales index should be flexible, encompass increasing and decreasing returns, and be conveniently parameterizable using management judgement. Flexibility is desirable so that managers' judgemental inputs will determine the general shape of the response function in any given situation. Decreasing returns, especially at high levels of ATE, may result from saturation effects of additional exposures among a highly exposed group of physicians who are poorer prospects for the product. Some empirical evidence of diminishing returns of market share to detailing expenditures may be found [16]. At low levels of exposure increasing returns seem plausible in that physicians may be more responsive after repeated exposure than they are likely to be after

only one or two exposures. Hence the response function should be able to represent increasing returns at low exposure levels and decreasing returns at high levels. This general S-shape was considered to be the most representative by the managers involved in this research. Finally, the function must be conveniently parameterizable by the manager's if it is to be used.

While a variety of functional forms would meet these criteria, including the S-shaped function used by Little in ADBUDG [13], we have used a simple cubic function in the present case. The sales index at t is given by:

$$SI(t) = \alpha + \beta[ATE(t)]^2 - \gamma[ATE(t)]^3 \qquad (5)$$

$$\text{If } \alpha + \beta[ATE(y)]^2 - \gamma[ATE(t)]^3 \leq 100$$

or

$$= 100, \text{ if } \alpha + \beta[ATE(t)]^2 - \gamma[ATE(t)]^3 > 100$$

where α, β, and γ are all non-negative constants. Since $0 \leq ATE(t)$, we have $0 \leq SI(t) \leq 100$. The upper bound of 100 is set to be consistent with (4) above. That is, no more than 100% of sales potential may be realized. Several alternative shapes of the response function are given in Figure 1.

Insert Figure 1 About Here

Notice in Figure 1 that one of the curves the maximum value of $SI(t) = 100$ at $\overline{ATE} < 1.0$. For that response function $SI(t) = 100$ (i.e., full sales potential will be realized) for any $ATE > \overline{ATE}$. This gives what we have termed a "holding" region for sales response to ATE. This behavior coincides with the managers' indications that for some drugs which have accumulated a considerable amount of exposure deleting detailing for one period would not affect sales. Use of the cubic form and the upper bound allow for this possibility.

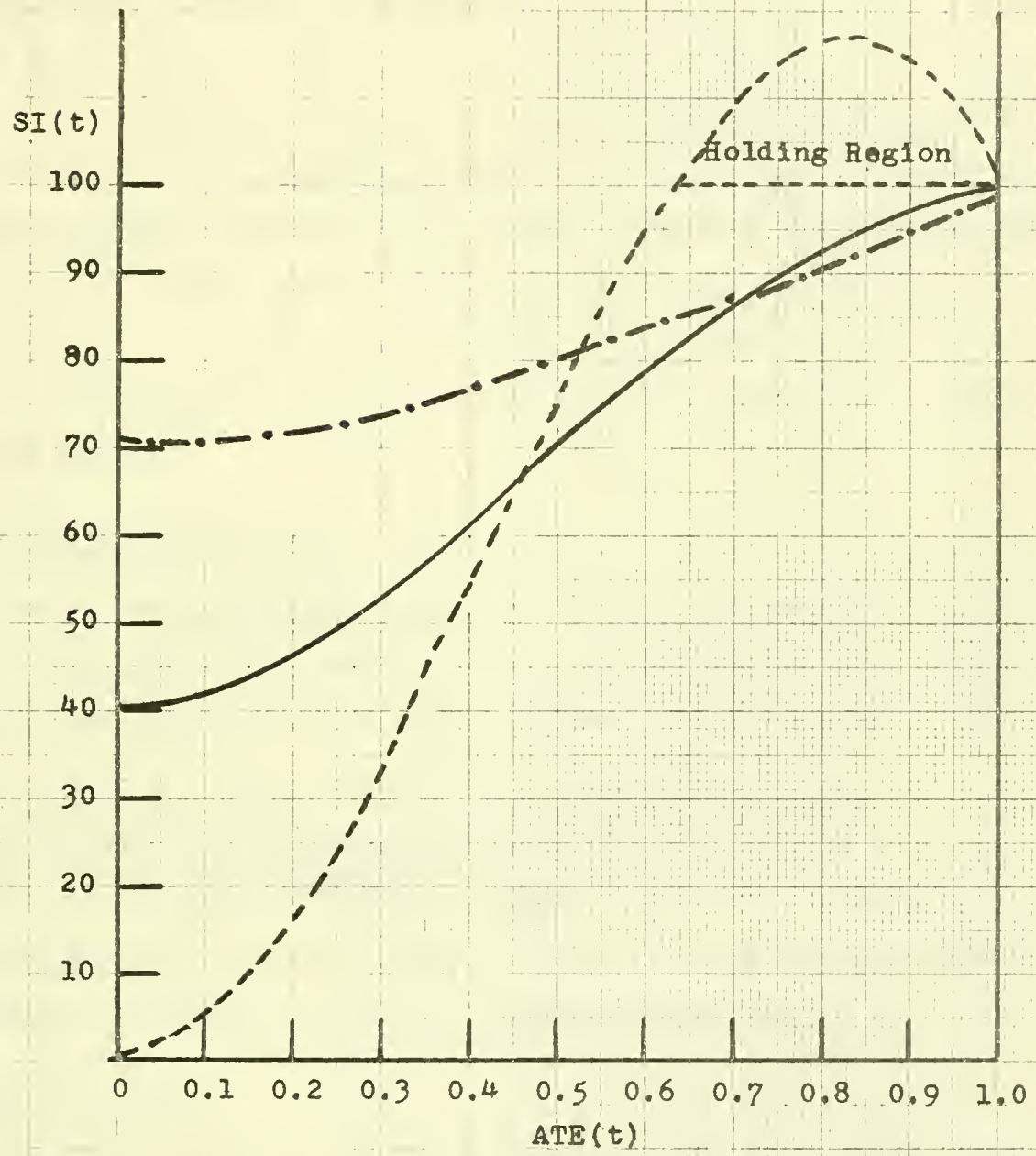

Figure 1. Some possible shapes of the function relating ATE(t) to SI(t).

4. PARAMETERIZATION

The product managers are asked a series of questions concerning each of their products. The answers to these questions are then used to infer parameter values in the model for each product.

Such questions which attempt to tap the experience and judgement of managers must be phrased in language clearly understood by them. Efforts were made to design questions in operational terms - i.e., with reference to events that the product managers might have observed or which they could readily imagine in terms of a hypothetical experiment. The actual interrogation and parameterization is performed via an interactive computer program.

4.1 Relative Exposure Value.

The first set of questions relates to the relative exposure value of alternative levels of detailing, which are c for complete coverage and q for quarter coverage. The questions used to elicit judgements on c and q were:

> Let us say that no detailing has a value of 0 and half coverage has a value of 1. Given the promotional materials and messages available next year for this product, estimate the value of quarter-coverage, e.g., 0.625. Similarly estimate the value of complete-coverage.

Note that the values are to be assessed conditional on what is to be communicated about the product to physicians over the planning horizon.

4.2 Reference Case.

The product manager is asked to forecast sales for his product in each of the next four quarters conditional upon messages and appeals which are feasible and upon some assumption about the detailing policy which will be pursued in each of these quarters. The product manager may choose any detailing policy he wishes to use in making the reference sales forecast. The conversational program suggests that he utilize a detailing plan with which he is familiar so that he

will find it easy to make the forecasts. The only constraint is that he not choose a reference policy of no detailing in each of the four quarters. The reason for this will become clear in the next section.

One use of the reference policy and reference sales is the development of sales potential. Given an initial value of ATE(0), an estimate of λ, and the parameters α, β, and γ,the reference policy may be used to generate an associated reference sales index. Sales potential for each of the first four quarters may then be estimated as:

$$SP(i, t) = \frac{RS(i,t)}{RSI(i, t)} \qquad (6)$$

where RS(i, t) and RSI(i, t) are the reference sales and the reference sales index for product i in period t, respectively. The reference case is also used in estimating the carry-over effect (or forgetting parameter), λ.

If the manager so chooses, he may develop a reference policy and reference sales for quarters beyond the fourth. Alternatively, he may specify an annual growth rate which is then used to extrapolate the sales potential for each quarter.

4.2 Estimating λ.

The parameter λ relates to the decay in ATE and hence the decay in SI and sales which will occur due to the wear-out or forgetting of past sales efforts. In order to estimate λ, we ask the product manager to forecast sales in the fourth quarter if the product was not detailed at all during the year. This forecast under no sales effort is then contrasted to the reference sales under the reference detailing policy in order to estimate λ. Our choice of sales in the fourth quarter as the basis for estimating λ deserves some explanation. We found that if we chose to use the second quarter, the product managers were very uncertain of their responses, However, if we asked for this fourth quarter accumulation of effects from no detailing, they were more comfortable in supplying a judgement.

The procedure used to calculate an estimate of λ utilized the following relationship:

$$SP(4) = \frac{RS(4)}{RSI(4)} = \frac{RSND(4)}{RSIND(4)} \tag{7}$$

where RSND(4) is the reference forecast of sales in the fourth quarter under a policy of no detailing during quarters 1-4 and RSIND(4) is the corresponding reference sales index under the no detailing policy. From (7) we have:

$$\frac{RS(4)}{RSND(4)} = \frac{RSI(4)}{RSIND(4)} = C \tag{8}$$

where C = Constant.

Hence:

$$RSI(4) - C * RSIND(4) = 0 \tag{9}$$

Now both RSI(4) and RSIND(4) are twelfth-order polynomials in λ. Consequently, numerical methods must be used to find λ between 0 and 1 which satisfies (9). One could utilize Newton's method or a simple numeric search on the feasible domain since the function is unidimensional. The latter method has been employed here.

4.4 Sales Index Parameters.

The cubic equation (5) which relates ATE(t) to SI(t) is parameterized by asking the following long-run questions:

> Suppose you were to use half-coverage for this product in each period for many periods. Consider the sales results after many periods under this policy of constant half-coverage. This will be considered 100 per cent of long-run, or constant half-coverage.
>
> Q1. What fraction of this long-run potential would be realized after many periods if the product were never detailed?
>
> Q2. What fraction would be realized after many periods if an "in and out" policy of half-coverage and no detailing were to be used in alternating periods?

The answers to Q1 and Q2 will be denoted by MIN and MED, respectively. Clearly one consistency condition which should be imposed is that $100 \geq MED \geq MIN$

We use the notion of a reference sales level of 100 under half-coverage for

two reasons. First of all, it enables the manager to focus on the relative effects of the alternative policies without becoming immersed in the problems of estimating the absolute level of sales in the distant future. Secondly, it relates the answers directly to SI values attained for specified levels of ATE.

Consider first the policy of constant half-coverage in each period. Since the REV of half-coverage was taken as 1.0, the asymptotic value of ATE will be unity. That is, since $0 < \lambda < 1$, for constant half-coverage:

$$\begin{aligned} ATE &= \lambda + \lambda(1-\lambda) + \lambda(1-\lambda)^2 + \ldots \\ &= \lambda \sum_{t=0}^{\infty} (1-\lambda)^t \\ &= 1 \end{aligned} \tag{10}$$

Similarly since REV=0 for no detailing, and since ATE is a weighted average of the EV's for the detailing policies used, we have ATE=0 for a constant policy of no detailing.

For the policy of alternating half and no coverage, the average long-run value of ATE will be 1/2. To see this suppose that half-coverage is applied in periods ..., n, n+2, ... and that no coverage is used in periods ..., n, n-1, ... Then we have for half-coverage in n:

$$ATE(n) = \lambda + (1-\lambda)\, ATE(n-1) \tag{11}$$

for no coverage in n:

$$\begin{aligned} ATE(n+1) &= (1-\lambda)\, ATE(n) \\ &= (1-\lambda)\lambda + (1-\lambda)^2\, ATE(n-1) \end{aligned} \tag{12}$$

and for half-coverage in n+2:

$$ATE(n+2) = \lambda + (1-\lambda)^2\, ATE(n) \tag{13}$$

Now in the long-run, the ATE's in periods having a corresponding level of detailing will be in equilibrium and hence ATE(n-1) = ATE(n+1) and ATE(n) = ATE(n+2). Using these results in (12) and (13), respectively, we have for no detailing:

$$ATE(n+1) = \frac{\lambda(1-\lambda)}{1-(1-\lambda)^2} \tag{14}$$

and for half-coverage:

$$ATE(n) = \frac{\lambda}{1-(1-\lambda)^2} \tag{15}$$

Then since half-coverage and no detailing occur on alternate periods, the average asymptotic value of ATE under this in and out policy will simply be the average of (14) and (15) which is:

$$ATE = \frac{1}{2}\ \frac{\lambda(1-\lambda) + \lambda}{1-(1-\lambda)^2} = \frac{1}{2} \tag{16}$$

Now since the long-run questions were phrased in terms of the fraction of long-run potential which might be realized under alternative policies, the estimates supplied by the managers are direct estimates of the sales index under these alternative policies. Then if these three estimates of the sales index are used in (5) along with their corresponding values of ATE, we have the following equations:

$$\begin{aligned} 100 &= \alpha + \beta - \gamma \\ MED &= \alpha + \beta\left(\frac{1}{2}\right)^2 - \gamma\left(\frac{1}{2}\right)^3 = \alpha + \frac{\beta}{4} - \frac{\gamma}{8} \\ Min &= \alpha \end{aligned} \tag{17}$$

which may be solved for estimates of α, β, and γ.

Two further comments about this function are in order. First, the cubic equation does not contain a linear term in ATE. The form used was found to give sufficinet flexibility without requiring the estimation of another (linear) parameter. The latter would have necessitated the asking of an additional long-run question. Further, since $100 = \alpha + \beta - \gamma$ and $ATE \leq 1.0$, the cubic equation will never exhibit negative returns to detailing over the admissible range of ATE values. In this context, negative returns are implausible.

4.5 Initial Accumulated Total Exposure

The model requires some estimate of ATE(0). Once the model has been

in use for a number of planning cycles, this will pose no problem in that the ATE value which resulted at the end of a previous run may now be used as ATE(0) in any current run.

Initializing ATE(0) the first time the model is used for a product is somewhat more problematical. One alternative is for the product manager to provide a direct judgemental estimate of its value. However, since ATE is an unobservable model construct, managers will be hard pressed to come up with a reasonable figure. An ability to supply direct judgements to the interactive program will, of course, prove useful in assessing the sensitivity of the analysis to ATE(0). The second major alternative is to utilize the detailing plan from past periods to initialize ATE. A simple approach is to compute the average REV from detailing in the past few periods as an estimate of ATE(0). The user may want to specify the values of c and q which held in these past periods if he has reason to believe they differ from the current values. One limitation of the average past REV approach is that it ignores the time pattern of past detailing in generating ATE(0). The problem is that to provide proper weights to past detailing requires that we know λ. Yet we estimated λ using a value of ATE(0). One way out would be to begin with an initial estimate of ATE(0) by using the average REV approach. This value of ATE(0) may then be used to obtain an initial estimate of λ. This initial λ value may then be used to refine the estimate of ATE(0) and the process may be repeated until two successive iterations show little difference in ATE(0) or λ. In the application discussed below, use of the simpler average REV approach was found to be adequate.

4.6 The Form of the ATE Function

Recall that the form of the ATE function (3) was taken to be:

$$ATE(t) = \lambda f(X_t) + (1-\lambda)\ ATE(t-1) \qquad (18)$$

which means that ATE is an exponentially smoothed function of current and past relative exposure values from detailing activities. A more familiar alterna-

tive formulation would be:

$$ATE(t) = f(X_t) + (1-\lambda)\ ATE(t-1) \qquad (19)$$

In this work, (18) was chosen because it greatly simplifies computations. If (19) were used, then the asymptotic value of ATE under a policy of constant half-coverage would be $1/\lambda$. Hence the forgetting parameter λ will be involved in the calculation of α, β, and γ in (5). But we need α, β, and γ to calculate λ numerically from (9). While an iterative scheme could be proposed, it would greatly complicate the computations. Since the response model is designed to run interactively, this is an important consideration. Note that use of (18) does not distort the relative weights of current and past promotion. The asymptotic values of ATE from (18) is just λ times those from (19). Since the values REV's form a ratio scale, multiplication by a constant is a permissable transformation.

5. ALLOCATION OF SALES EFFORT ACROSS THE PRODUCT LINE

The allocation of sales effort across products over time is accomplished by a simple heuristic method. The objective of the heuristic is to find changes in a current detailing plan which will lead to an improved total profit contribution for the product line as a whole.

The decision variables X(i,t) are the levels of detailing to allocate to each product i during each time period t. Recall from our earlier discussion that the company considers detailing in four discrete amounts: no detailing, quarter-coverage, half-coverage, and complete-coverage. By convention, we define quarter-coverages as one unit of detailing. Then since half-coverage requries twice as many calls, half-coverage involves two units of detailing. Similarly, complete-coverage uses up four units of detailing, while no detailing clearly uses no units. Hence, in the current application we have $X(i,t) = 0, 1, 2, \text{ or } 4$ for $i = 1, \ldots, I$ and $t = 1, \ldots, T$.

Recall that in the present case, the company's long experience in the pharmaceutical market led them to limit the number of drugs promoted to an individual doctor on any given call to three. Hence, the detailing policy for the product line in any period t must be less than or equal to the available units of detailing. Since the focus of the present model is upon the allocation of available sales effort and not upon the issue of the size of the salesforce, the detailing policy in each period must satisfy the equality constraint:

$$\sum_{i=1}^{I} X(i,t) = 12. \tag{20}$$

If for some reason the company wants to make a preemptive allocation of detailing effort to some of its products, it may reduce the available 12 units by the appropriate amount and then remove that product (or products) from the set considered by the model. This might occur in the case of a new product which has limited sales potential but which may help to establish the company's

technical reputation in important therapeutic areas. The detailing would then be oriented toward developing over all long-run company reputation in the medical community and should not then be required to face the test of profitability. Of course, the model could be used to help management assess the foregone profit implications of such a strategy.

The objective or evaluation function which is used is total product line contribution over the planning horizon

$$TGP = \sum_{i=1}^{I} GM(i) \sum_{t=1}^{T} SP(i,t) * SI(i,t) \qquad (21)$$

where TGP = total gross product line profits over the planning horizon

GM(i) = % gross margin on product i

and recalling that

$$SI(i,t) = MIN \{100, \alpha(i) + \beta(i) [ATE(i,t)]^2 - \gamma(i)[ATE(i,t)]^3\}$$

$$ATE(i,t) = \lambda(i) f_i[X(i,t)] + [1 - \lambda(i)] ATE(i,t-1)$$

and

$$0 \leq ATE(i,t) \leq 1.0$$

Time period interdependencies result from the fact that a detailing decision X(i,t) in period t for product i will affect the product's ATE and hence the fraction of realized sales potential and gross margin contribution in periods t + 1, t + 2, ..., T. Hence, the policy decisions in any period must consider the future profit implications as well as the current period results.

The allocation is accomplished by a heuristic algorithm which is described in the Appendix. Briefly, the allocation begins with some initial feasible allocation of effort to each product for each period in the planning horizon. This could of course be the null solution of no detailing for any product in any period, but in most cases it will be advantageous for comparison purposes to use an initial plan developed by management. A plan specifies the amount of sales effort to allocate to each product in each period.

Computation begines by zeroing out the plan in period 1 for each product. A marginal analysis is then performed for each product taking account of the future impact of sales effort in t=1 given the current plan in periods 2, ..., T. Effort is allocated to products based upon the marginal response until total sales time is exhausted. Attention is then turned to period 2, only with the input values of ATE(i, 1) which result from the just completed allocation in period 1. Now the plan is zeroed in period 2 and the process is repeated with attention once again being given to future efforts. This process is repeated to the end of the planning horizon. When this procedure results in the same plan for two successive passes over all T periods in the planning horizon, the procedure terminates.

While the problem could be formulated as a dynamic programming problem, computational limitations restrict its utility to small scale problems. This heuristic method has been found to be adequate in that it is fast and has led to an improvement of the managers' plan in test cases. The current operating version of the program will handle up to 50 products over planning horizons up to 16 periods and for up to 10 discrete levels of sales effort.

6. DISCUSSION OF AN APPLICATION

Before presenting some results from the initial application of this system, it will be helpful to describe the manner in which this allocation problem had previously been handled in the firm studied and to outline the approach followed in developing and implementing the present model.

6.1 Model Development and Implementation Procedure

The stimulus for this work was an earlier investigation which involved an econometric analysis of market response to expenditures for various types of pharmaceutical promotion [16]. The results of that study raised a number of questions about the planning and control of the firm's detailing activities. The decisions process within the firm which resulted in detailing effort being allocated to products began with the preparation of a marketing plan for each drug by a product manager. Detailing was considered to be the most effective way of influencing doctors and plans for using other forms of promotion such as journal advertising or direct mail were developed so as to supplement the detailing allocation a product received. Preliminary recommendations for the detailing of a drug for each period in the coming year were passed on to the group product manager. Typically, the total amount of detailing effort so requested for all products in the line would exceed the capacity of the available field force. Compromise allocations and scheduling conflicts would then be worked out by informal analysis and negotiations among the product managers and their supervisor. Lacking any systematic approach to measuring the effectiveness of detailing, management emphasized certain rules of thumb (expense to sales ratios) as a means of controlling expenditures on promotion. For this purpose, detailing expense was allocated to different products in the line even though it represented an essentially fixed cost. As might be expected, these practices had the effect of making it easy to rationalize detailing support for drugs already producing large amounts

of revenue but difficult to justify allocations to products with small sales volumes. In general, there was little appreciation of the notion that there were opportunity costs associated with applying detailing effort to certain products but not others.

Prior to the initiation of this project, the product managers in this firm had had little exposure to modeling concepts and no experience at all with interactive systems. It was recognized from the beginning that the product managers would have to be involved in the development of the model if they were to gain the level of understanding required for them to accept and use it. Furthermore, in order to develop a model which would take advantage of, and faithfully portray their knowledge and views about market response to detailing, they had to be willing to devote time to enlightening the model builders about these matters. The dual task of educating both themselves and others represented a significant intrusion into the product managers' already busy schedules. Hence, obtaining their cooperation and support was critical.

Lacking much familiarity with models, the product managers initially found the announced purposes of the project vague and were skeptical about its likely utility. The situation called for a way of making the goal to which they were being asked to commit themselves more concrete and meaningful. In an effort to accomplish this, the first step taken in the project was to give the product managers an opportunity to see and use an existing interactive marketing model. The system employed for this demonstration was a simple version of Little's advertising budgeting model, ADBUDG [13]. A day-long session was held during which the product managers first worked at a terminal and then discussed the model at length. The experience impressed them and by the end of the day they were discussing with enthusiasm the possibilities for dealing with their own operations in a similar way.[7]

Following this brief educational effort, development of the response model commenced. A series of lengthy discussions were held with the product managers and a small number of other knowledgeable executives for the purpose of uncovering their implicit model(s) of how detailing affects physicians' prescribing behavior. Once the behavioral phenomena management considered important were identified and understood, they had to be represented and interrelated in a formal model structure and methods found to parameterize the model with answers to questions that the product managers were willing and able to answer. This proved to be the most difficult and time-consuming phase of the project as evidenced by the fact that ten weeks were spent structuring the model. The interactive programming was accomplished in two weeks. A number of different approaches were tried and discarded before a formulation was hit upon that the managers found both complete and operational and which appeared sensible and internally consistent from a modeling standpoint. As problems were encountered and solutions proposed, the product managers were frequently called upon to clarify, check and test various issues and ideas. The frequent interaction kept the product managers informed about how the project was progressing and enabled them to see how their inputs were influencing the process. The contact with the product managers also served to discipline the model building effort by requiring that management understanding and involvement be maintained.

6.2 Results

After becoming familiar with the interactive response model, the product managers used it to evaluate alternative detailing policies for the products assigned to them in the coming year. This provided estimates of the response parameters required as input for the allocation heuristic. The allocation program was then run to obtain a recommended allocation of the available detailing resources among the set of products considered, given the product managers'

judgements about how effective various detailing policies would be during the next year. The run was made for nine products (handled by three managers) and eight quarters (or two calendar years).[8] While our primary interest was in allocating detailing for the next four quarters, a two year or eight quarter planning horizon was used in order to diminish end effects. The allocation recommended by the model was then compared with the detailing plan which the managers had developed earlier without the assistance of the model. The results are presented in Table 2. For this problem the allocation heuristic required 15 seconds of time on a 360/67.

Insert Table 2 about here

The estimated profit improvement associated with using the detailing plan generated by the model rather than the one management had arrived at previously amounted to $86,000 over a one year period. The comparable figure for the entire two year planning period was $139,000. Thus it may be seen that the estimated first year increment was not merely the result of borrowing future profits but rather, if anything, it tends to underestimate the value of the longer-run benefits. This indicated profit improvement in the first year exceeds our estimate of full commercial development costs by more than three times.

Significantly, the detailing allocation that produced this profit gain called for a few major alterations in management's original plan rather than numerous small changes in it. For six of the nine products, there was little or no difference between the two plans--in regards to either the amount of detailing

TABLE 2

A Comparison of Management
and Model-Based Allocations

PERIOD	Products																	
	1		2		3		4		5		6		7		8		9	
	I	M	I	M	I	M	I	M	I	M	I	M	I	M	I	M	I	M
1	Q	Q	H	N	H	C	N	N	H	N	Q	C	H	Q	N	N	N	N
2	H	Q	Q	Q	H	Q	H	N	Q	Q	Q	C	Q	Q	N	N	N	Q
3	H	H	Q	N	Q	Q	H	N	Q	Q	H	Q	Q	C	N	N	N	Q
4	H	H	Q	Q	Q	Q	H	N	Q	Q	H	H	Q	H	N	N	N	Q
5	H	H	Q	Q	Q	Q	H	N	Q	Q	H	H	Q	Q	N	N	N	H
6	H	H	Q	Q	H	H	Q	N	Q	Q	H	H	Q	Q	N	N	N	Q
7	H	H	Q	Q	Q	Q	H	N	Q	N	H	C	Q	Q	N	N	N	Q
8	H	H	Q	N	Q	H	H	N	Q	Q	H	H	Q	Q	N	N	N	H
Total & Period Profit	1559	1552	906	905	918	925	622	463	1623	1632	4192	4304	511	523	650	650	1179	1355
Model Profit Minus Initial Profit	-7		-1		+7		-159		-1		+112		+12		0		+176	

I Denotes Managers' Initial Plan

M Denotes the Model's Allocation

C = Complete Coverage
H = Half Coverage
Q = Quarter Coverage
N = No Coverage

support each product would receive or the manner in which it was scheduled. However, the other three products considered were affected markedly. What the model-based plan essentially did was withdraw detailing entirely from one product to which management had assigned considerable support and re-allocate it to two other products, one of which had not received any detailing whatsoever in management's original plan. The drug which the model deleted from the detailing plan had not acheived the sales level hoped for in the past and developing effective promotional messages for it had proven to be problematical. Thus, it appeared that management's desires rather than their realistic expectations had influenced its inclusion in the original plan. Replacing this latter drug in the model's plan was one which had never been detailed previously. The drug's specialized nature and relatively small sales volume had led management to assign it a low priority when evaluating the detailing potential of the various products in the line. After analyzing the situation more carefully in the context of model, management judged the drug's sales to be sufficiently responsive to detailing to warrant receipt of quarter-coverage in several periods. The other important difference between the two plans involved the drug currently producing the largest dollar volume of all those considered. Here, the model recommended substantially more detailing than management had allocated to it for the comming year.

Management's original plan and that developed by the model differed not only in the amount of detailing allocated to the three products mentioned above but also with respect to the frequency with which the various detailing policies were employed. Summing across products and time periods, management had used the quarter-coverage policy 30 times, the half-coverage policy 25 times, and complete coverage not at all. In contrast, the corresponding figures for the model-based plan were 30 for quarter-coverage, 15 for half-coverage, and 5 for complete coverage. Although quarter-coverage was applied equally often under both plans, the model utilized the maximum or complete coverage policy more and the intermed-

iate or half-coverage alternative less than management had. The fact that a complete coverage policy never appeared in management's plan appears to be the consequence of the bargaining which the managers engaged in to arrive at an allocation. As was noted earlier, the requests for detailing support initially made by the product managers for their drugs would generally have to be adjusted downward in order to satisfy the constraint on the amount of detailing time available. Faced with the need to cut back on the amount of detailing requested for various products, a natural way to proceed was to argue that a product could get by with something less than the maximum allocation available. Hence, the process of negotiation and compromise tended to work against the employment of complete coverages.

A second application of the model was made in a different division of the company. This division also sold ethical drug products, but an entirely different sales force was used and it covered an entirely different geographic region. The policy alternatives in terms of the levels of coverage considered were different in this case. The results, however, were qualitatively similar. Once again only a few products were affected in a major way and the indicated increased profitability was of the same order of magnitude relative to the profitability of the managers' initial plan.

7. CONCLUSION

Two questions are often raised about decision calculus models: (1) What is there to prevent managers from manipulating the input to obtain whatever preconceived result they wish, and (2) How does one know if the model is "right?" Both queries reflect a disquieting suspicion if not outright distrust of the use of judgemental data to parameterize such models. Little [13] has discussed these issues and here we will make only a few additional comments about them suggested by experience gained in the present study.

While the model described above could easily be gerrymandered, we have observed no tendency for managers to be perverse in this regard. To the contrary, having struggled with the problem of allocating detailing effort across products for some time, they were genuinely interested in finding ways to improve their decisions in this area. Furthermore, certain features of the product management organization served as safeguards against the model being abused. The system requires that each product manager make his assumptions about market response to detailing quite explicit. The managers were certainly aware that their judgements would directly affect the allocation but they also realized that the inputs they provided would be scrutinized by others and therefore, must be defensible. The firm had the policy of regularly rotating the drugs assigned to different product managers so that over time each would become familiar with the entire product line. Thus, any product manager could be certain that the reasonableness of the input he supplied for the model would be evaluated by his fellow managers who were not only knowledgeable about the same products but also had a vested interest in how the final allocation was made. For these reasons, it is unlikely that an attempt to advance one's own cause by rigging the input would go undetected.

"Validating" or evaluating decision calculus models is a more complicated matter. For example, the profit improvement figure cited above associated with use of the model-based plan was of sufficient absolute magnitude to be worthwhile but would represent a rather small relative increase--less than five per cent of the previous year's profits. Such a change would fall well within the range of past variability of profits and as a result, would be difficult to verify. The ability of the model to reproduce past history and forecast the future are important tests of fidelity that can be made. Given that the model is so overparameterized, such global assessments tend to be of limited value for pinpointing and correcting estimation or structural problems and weaknesses. For these purposes, special

studies and empirical estimates of parameters are required. Such needs may stimulate the evolution of measurement programs and more complex data-based models. However, what often leads the analyst to consider developing this type of model in the first place is that measurement has proved to be problematical and/or expensive, Even in such cases where the possibilities for extensive empirical study and evaluation are remote, there is still reason to believe that the use of simple models that elicit and process judgements can be productive. Significant gains may be realized in two ways. First, by decomposing a decision problem and asking a manager to make judgements about the component parts, one attempts to take full advantage of his experience and knowledge of the details of the situation. Secondly, by using the model to organize the various elements of the problem and set forth the consequences implied by the analysis, one seeks to eliminate inconsistencies and relieve the manager of computational tasks.

The argument that decisions can be improved by a model which interrelates a set of judgements in a consistent fashion is certainly not new and some evidence to support it does exist. Some time ago, Bowman [2] was able to show that important cost savings could be realized by applying production and employment scheduling decision rules where the coefficients were inferred from management's own past behavior rather than a statistical cost function.[9] Bowman suggested that managers are sensitive to key decision criteria but imperfect information processers. Although erratic, their decisions tend to be within the range of "preferred alternatives." It follows then according to Bowman, that decisions can be improved by taking the manager's coefficients and applying them in a more consistent manner. An example from marketing that seems to substantiate Bowman's views is provided by Edelman [6], who was able to compare the outcome of decisions made "with and without" a model--something that it is rarely possible to do meaningfully. A model was used to generate competitive bids for seven cases using judgemental estimates of the response parameters supplied by managers.

Edelman also reports the bids the same managers had arrived at by using their customary decision making procedures. In all seven cases, the price bids suggested by the model were superior to those developed by the traditional method. Ultimately of course, the quality of the judgements will determine what gains can be achieved but Edelman's work does serve to illustrate the value of the functions performed by a model.

An integral part of the decision calculus concept is the notion that model development should be evolutionary--one begins with a simple structure and gradually moves to more complex versions [18]. A complete marketing planning system for this problem situation would incorporate the effects of direct mail and journal advertising and competitors' promotional activities. Advertising and competitive effects can only be represented indirectly in the model described here be modifying forecasts and parameter values. Allocating detailing effort without taking explicit account of advertising parallels the firm's practice of making a decision on detailing first and then using other forms of promotion for supplementary support. It would be desirable to treat these considerations in a more refined way. In many circumstances where salesmen promote several products, interdependencies among items in the line are likely to be important. The type of approach proposed by Urban [17] might be used to handle such problems.

APPENDIX

Outline of the Allocation Heuristic

Step	Operation
1	Read in information common to all drugs. Number of periods in plan, available detailing strategies, and detailing units available by period.
2	Read in information specific to each drug. Name of the drug, sales potential SP(i,t) for t = 1, 2, ..., T; gross margin, GM(:); initial accumulated total exposure, ATE(i, t = 0); the forgetting constant, $\lambda(i)$; and the parameters of the cubic function, $\alpha(i)$, $\beta(i)$, and $\gamma(i)$.
3	Read in initial detailing plan for each product in each period. Check that Plan does not exceed amount of detailing effort available. This plan is the initial current plan. It is also the previous plan for the first pass.
4	Print out an evaluation of the initial plan. List the detailing strategies by product and time period. List the sales and profit results by product under the initial detailing plan.
5	Set time period index t = 1.
6	Set X(i, t) = 0 for i = 1, 2, ..., I.
7	Set $\sum_{i=1}^{I} X(i, t) = 0$
8	For each product i = 1, 2, ..., I compute the incremental profit which would result in the current and future periods from an increase of detailing on product i to the next highest level. Call this IP(i, t, $\Delta X(i, t)$) where $\Delta X(i, t)$ is the increment in detailing units involved in increasing detailing to the next highest level. This calculation is made subject to the constraints that $X(i, t) \leq 4$ for all i and t and that $\Sigma X(i, t) \leq$ DAVAIL(t), where DAVAIL(t) is the amount of detailing available in t.
9	Select the product i having the largest incremental profit, IP(i, t, $\Delta X(i, t)$). Call this product i_1. If $\Delta X(i_1, t) = 1$ and increment $X(i_1, t)$ by $\Delta X(i_1, t)$ and go to step 12. Otherwise, go to step 10.
10	Select the products having the second and third largest IP and for which $\Delta X(i, t) = 1$. Call these products i_2 and i_3, respectively.
11	If $IP(i_1, t, \Delta X(i_1, t)) \geq IP(i_2, t, \Delta X(i_2, t)) + IP(i_3, t, \Delta X(i_3, t))$, then increment $X(i_1, t)$ by $\Delta X(i_2, t)$ and go to step 12. Otherwise, increment $X(i_2, t)$ by $\Delta X(i_2, t)$ and go to step 12.

12 If $\sum_{i=1}^{I} X(i, t) \leq DAVAIL(t)$, go to step 8. Otherwise go to step 13.

13 If $t < T$, increment t by one and go to step 6. Otherwise go to step 14.

14 Compare current plan to previous plan. If identical go to step 15. If not, set previous plan = current plan and go to step 5.

15 Print out allocation to products by time period. In addition, print out sales and profit estimates for each product by each time period under this allocation.

STOP.

FOOTNOTES

[1]The present treatment of the sales effort allocation problem assumes that control over the sales force's activities is centralized to a considerable degree. Rather than have management specify which products salesmen are to promote and how much, it may be desirable to give salesmen more freedom to decide for themselves how best to utilize their time. Some flexibility is needed in order for the salesmen to take advantage of their detailed knowledge of the market. One means of achieving decentralized control is to tie salesmen's compensation to the profitability of the various items they sell. Farley has made an interesting analysis of this policy [8, also see 5]. He derives an "optimal" commission rate structure for a product line which assures that if a salesman allocates his efforts to different products so as to maximize his own income, then total company profits will also be maximized.

[2]Little has developed an advertising budgeting model possessing these characteristics [13]. Edelman's pricing models represent earlier work of a similar nature [6, 7]. More recently, Lodish formulated a decision calculus-type model to aid salesmen in call planning [14].

[3]See [3, 4] for examples of experimentation in this field.

[4]The response function is, of course, parameterized for each product separately. To simplify the notation in this section, we have not carried a product subscript.

[5]The above weighting scheme was chosen because it was compatible with the views of product managers with whom we were dealing here. In other settings, it would probably make more sense to assign values of zero and one to the policies of no detailing and complete or maximum coverage, respectively.

[6]See [9] for a review of different approaches to this problem.

FOOTNOTES (Cont.)

[7]Although ADBUDG deals with budgeting advertising for a consumer good, it worked well as a training aid for these product managers from the pharmaceutical industry. Consumer advertising is represented in a simple manner that appeared similar enough to detailing for these product managers to recognize that the basic concepts and capabilities of the system were relevant to their own field but, at the same time, the problem context was sufficiently unlike their own situation that it did not appear threatening or unrealistic and provoke defensive and critical reactions that might have impaired further interest. The study of problems involved in implementing models and the development of means of overcoming them such as training aids are subjects which deserve much more attention than they have received. For an example of an interesting beginning effort in this area, see [10].

[8]As noted earlier, the company had ten products which it considered candidates for detailing support. In the present application, management chose to assign half-coverage in each period to one of their drugs. Hence, the allocation was made across nine products with ten rather than twelve units of detailing available for allocation.

[9]See [11] for some more recent work in the same spirit.

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